Elementary Proof of Cauchy's Theorem.

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Denote the sides of a right triangle (Argand Diagram) by x, iy, z. Multiply each of these by the complex number w (plane vector) giving $wz = wx + wiy$. Hence the proposition: *If on the three sides of a right triangle, similar and similarly placed triangles be constructed, then the sum of the corresponding sides (considered as vectors) of the leg triangles is equal to the corresponding side of the hypothenuse triangle.*

At the limit this becomes

$$dW = wdz = wdx + widy = w(dx + idy).$$

Hence *the change in the function W is the same whether z follows the elementary paths dx, idy, or the resultant of these, dz.*

Hence $W = \int dW$, the sum of all these changes in W due to changes in z, will be the same whether z follows the elemental paths Σdx, Σidy or resultants of these, so long as in the deformation of the path of z into Σdx and Σidy we do not pass over any point in which $w = \infty$.

Hence W *always attains the same value for the same value of z, whatever the path of z, provided no point where $w = \infty$ is enclosed between the paths.*

Since we can take the end of the path of z as near the beginning as we choose, or coincident with it, we can say:

(Cauchy's Theorem) $W = \int dW = \int wdz$ *taken around a closed curve enclosing no point where $w = \infty$ is zero.*

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